

SMALL x AND SMALL Q^2

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Abstract

I discuss (i) hadron-hadron and photon-hadron total cross-sections, (ii) elastic scattering, (iii) the rôle of minijets in total cross-sections and the need to include them in Monte-Carlos for small- x structure functions, (iv) how the two pomerons affect small- x behaviour, and (v) rapidity-gap physics.

1. Total cross-sections

This talk is about nonperturbative effects at high energy, and this means that its central theme is Regge theory^[1]. I remind you that if one plots the spins α of families of particles against their squared masses t , one finds linear behaviour. The best example is the ρ, ω, f, a families, shown in figure 1. Regge theory tells us that if we extrapolate the line $\alpha(t)$ to negative t , so that t may then be interpreted as a momentum transfer, the sum of the exchanges of all the particles in any of the families contributes to a high-energy elastic scattering process at centre-of-mass energy \sqrt{s} and momentum transfer t as follows:

$$T(s, t) \sim \beta(t) \xi_\alpha(t) s^{\alpha(t)} \quad (1)$$

Here $\beta(t)$ is some unknown real function and $\xi_\alpha(t)$ is a definite phase calculated from $\alpha(t)$. We call such a sum of exchanges Reggeon exchange, and depict it diagrammatically by a jagged line as in figure 2.

The optical theorem allows us to calculate the total cross-section from the elastic amplitude:

$$\sigma^{\text{TOT}} = \frac{1}{s} T(s, t=0) \sim s^{\alpha(0)-1}. \quad (2)$$

According to figure 1, for the ρ, ω, f, a families $\alpha(0) \approx \frac{1}{2}$, so that their contribution to σ^{TOT} has behaviour close to $1/\sqrt{s}$. In order to obtain rising total cross-sections, as are found experimentally, we need to introduce another Reggeon, known as the soft pomeron. This has

$$\alpha(0) = 1 + \epsilon \quad (\epsilon > 0) \quad (3)$$

and in order to describe the data we need^{[2][3]} $\epsilon \approx 0.08$: see figure 3.

The soft pomeron is nonperturbative in origin, and is almost certainly a consequence of nonperturbative gluon exchange^[4]. It would be nice to think that actually it is associated with glueball exchange, so that one can make a plot something like figure 1 but with glueballs instead of quark states. As I shall explain, near $t=0$ the slope of the pomeron trajectory is somewhat smaller than for ρ, ω, f, a :

$$\alpha(t) = 1 + \epsilon + \alpha' t \quad (4)$$

with $\alpha' = 0.25 \text{ GeV}^{-2}$ instead of the $0.8 = \text{GeV}^{-2}$. If $\alpha(t)$ remains straight for positive t , there should be a 2^{++} glueball with mass about 1900 MeV. If glueballs are not found, then pomeron exchange is probably just the exchange of a number of separate nonperturbative gluons.

The other pomeron, the Lipatov pomeron^[5], corresponds to perturbative gluon exchange. It has a significantly larger value of ϵ , about 0.5. There is no firm evidence that it has yet been seen in any experiment, though there is the hope that the HERA measurements of νW_2 at very small x may at last have revealed it. Maybe, just maybe, it has even been seen in the $\bar{p}p$ total

cross-section measured at the Tevatron^[6]. In figure 3, I show the published $\sqrt{s} = 1800$ GeV measurement of the E170 experiment. The CDF experiment this summer has reported a much larger value, $\sigma\bar{p}p = 80.6 \pm 2.3$ mb. If it should turn out that the conflict between these two results is resolved in favour of CDF, it might indicate the onset of Lipatov-pomeron exchange.

2. Elastic Scattering

The form (1) may be compared with elastic-scattering data. Long ago^[7], the ISR pp data yielded the value $\alpha' = 0.25$, and this then led to remarkably successful predictions^[3] for $\bar{p}p$ elastic scattering at the CERN collider and at the Tevatron: see figure 4. This is a real triumph for Regge theory: between ISR and Tevatron energies the change in the forward elastic slope is more than 3 GeV^{-2} and is a direct confirmation that the value of α' is correct. Likewise, the phase of the forward amplitude is well predicted, now that the UA4 collaboration has changed its result at CERN collider energy^[8].

I should mention that we know, from unitarity, that it is not enough to consider just the exchange of a single pomeron. One must add in also the simultaneous exchange of two or more pomerons. Even after more than 30 years of Regge theory, we do not have any certain way to calculate these higher exchanges. Unlike other authors, Donnachie and I believe^[3] that their effect at present energies is small in the forward direction (less than 10%), though they are essential away from $t = 0$.

3. Minijets

I have shown the successful prediction from Regge theory for the γp total cross-section at HERA. There were some other predictions that were much larger, and it is interesting to discuss why they turned out to be wrong.

They were mostly based on perturbative calculations of inclusive minijet production. Since the scale of nonperturbative effects is typically 1 GeV, I would expect that perturbation theory reproduces the inclusive cross-section $d\sigma/dp_T$ reasonably successfully (say, to within better than a factor of 2) down to $p_T^{\min} \approx 1$ GeV.

Now^[9]

$$\int_{p_T^{\min}} dp_T \frac{d\sigma}{dp_T} = \langle n \rangle \sigma(p_T > p_T^{\min}) \quad (5)$$

where $\sigma(p_T > p_T^{\min})$ is the contribution to σ^{TOT} from events containing minijets, and $\langle n \rangle$ is the average number of minijets in these events. One calculates $d\sigma/dp_T$ from the familiar diagram of figure 5a, which involves the structure functions of the incoming particles and a hard scattering. The naive expectation from figure 5a is that exactly 2 minijets are produced, that is $\langle n \rangle = 2$ in (5). However, if one calculates the left-hand side of (5) and compares with the HERA measurements, this would require $\sigma(p_T > p_T^{\min}) > \sigma^{\text{TOT}}$, which is wrong by definition.

So it must be that $\langle n \rangle > 2$. This is not really a surprise: it has long been known that it is correct to use figure 5a to calculate the inclusive cross section $d\sigma/dp_T$, but^[10] that this simple figure cannot be expected correctly to reproduce other features of the event structure. One way in which $\langle n \rangle$ can be greater than 2 is from multiple parton-parton scatterings involving two or more partons from each of the initial particles. I would expect this to be very important in high-energy nucleus-nucleus reactions^[11], but I doubt whether it is the main mechanism in γp collisions.

Rather, there is another mechanism^[9] which is intrinsically nonperturbative and which has so far not been included in the Monte Carlos that are used for HERA data analysis. In order to calculate $d\sigma/dp_T$ from figure 5a, one needs the two structure functions down to fractional-momentum values x of order $x \sim p_T^{\min}/\sqrt{s}$, that is very small values when \sqrt{s} is large. But, from elementary kinematics, when a parton of momentum k is pulled out of a particle of momentum

p with $k = xp + \dots$, the squared invariant mass of system it leaves behind is^[12]

$$s_0 \sim -\frac{k^2 + \mathbf{k}_T^2}{x} \quad (6)$$

and so is large when x is small. That is, the upper and lower clusters of residual fragments of the initial hadrons in figure 5a each have large invariant mass and so they are very likely to contain additional minijets.

Another way to understand this is from a ladder model for the total cross-section: figure 5b. Of course, one must integrate over the transverse momentum round each loop of the ladder. Consider the contribution from a “hot” loop somewhere in the middle of the ladder, that is from large transverse momentum in that loop. If I cut the ladder down the middle and draw circles round the lines above and below the hot loop, as I have done in the figure, the left-hand side of the diagram has exactly the form of figure 5a. (Including also the right-hand side just squares it, as is needed to calculate the cross-section.) However, it may well be that another loop of the diagram is hot too, or even several other loops, corresponding to several pairs of minijets. As is well known^[5], at small x there is no k_T ordering, so the hot loops are most likely not next to one another, but rather are separated by loops that are not hot, so that one cannot calculate the multi-minijet production purely from perturbation theory.

Consider now νW_2 at a value of Q^2 that is only moderately large, so that perturbative evolution has not yet set in and the parton model applies: figure 6. The squared energy of the lower bubble, which is the amplitude for finding a parton k in the proton p , again satisfies (6) and so is large at small x . Since this bubble is an elastic strong-interaction amplitude, its high-energy behaviour is governed by Regge theory and is just a sum of terms $s_0^{\alpha(0)}$. If we insert this in the calculation of νW_2 from figure 7, we obtain^[12] a sum of terms $(1/x)^{\alpha(0)-1}$.

It is well known that νW_2 contains such Regge terms at small x , approximately a constant from soft-pomeron exchange and close to \sqrt{x} from f, a exchange. What is not so well known is that such a behaviour, which I stress is nonperturbative, arises because the proton fragments that remain when a small- x parton is pulled out have large invariant mass.

The Monte Carlos used at HERA set both k^2 and k_T to zero (before the perturbative evolution begins) and so do not make s_0 large. Thus they miss a nonperturbative effect which is surely important.

4. The two pomerons

As I have explained, at least at moderate Q^2 the small- x behaviour of νW_2 should contain the same powers of $1/x$ as the powers of s that appear in the fit of figure 3 to the γp total cross section. Another important feature of νW_2 is that, at $Q^2 = 0$, it vanishes linearly with Q^2 . Indeed,

$$\sigma^{\gamma p} = \frac{4\pi^2\alpha}{Q^2} \nu W_2 \Big|_{Q^2=0}. \quad (7)$$

The simplest fit to the small- x data that has these features is provided for by the form

$$\nu W_2 = X x^{-0.0808} \left(\frac{Q^2}{Q^2 + a^2} \right)^{1.0808} + Y x^{0.4525} \left(\frac{Q^2}{Q^2 + b^2} \right)^{0.5475}. \quad (8)$$

Figure 7 shows the result of such a fit^[13] to the small- x NMC data up to $Q^2 = 10 \text{ GeV}^2$. It is a two-parameter fit, because for each choice of X and Y one determines a and b in such a way as to retrieve, through (7), the γp fit shown in figure 3. The best-fit values are $a = 750 \text{ MeV}$ and $b = 110 \text{ MeV}$.

If one sets $Q^2 = 8.5$ and extrapolates the fit down to $x = 2 \times 10^{-4}$, one obtains $\nu W_2 \approx 0.6$. This is much smaller than the measured value^[14] of about 1.4 ± 0.5 reported by H1, which raises

the exciting possibility that the discrepancy may be attributed to the presence of the second pomeron, the Lipatov pomeron. In order to decide whether this is true, it is necessary to learn how the two pomerons live together. There are those who believe that, as Q^2 increases, the soft pomeron goes smoothly over to the Lipatov pomeron, that is $x^{-0.08}$ changes to something like $x^{-0.5}$. However, I think it is more likely that the two terms should be added together.

This is because of a simple model of the Lipatov pomeron that Collins and I studied a couple of years ago^[15]. We wrote the Lipatov equation^[5] as

$$T = T_0 + K \otimes T \quad (9)$$

where the last term denotes, as usual, a convolution of the amplitude with the Lipatov kernel K that includes an integration over transverse momentum k_T . The kernel K is calculated from perturbation theory, and so it is not valid to use it below some value Q_0 which is of order 1 GeV. So we restricted the integration to $k_T > Q_0$. In order to include nonperturbative effects associated with smaller values of k_T , we chose the driving term T_0 in the equation to correspond to soft-pomeron exchange. In the generalised ladder-like diagrams that give the Lipatov equation, this amounts to restricting the small k_T to the part of the ladder near to the proton, and taking all the transverse momenta nearer to the photon to be perturbative. This cannot be more than an approximation — I have already remarked that, at small x , there is no k_T ordering — but it is better than not including nonperturbative effects at all. In order to be able to solve the Lipatov equation exactly, we approximated the kernel K by a simple function that has the essential features of the true kernel. The resulting output amplitude T was a sum of terms corresponding to the Lipatov power $x^{-0.5}$ and the input driving term $x^{-0.08}$

We studied^[15] another important effect with our equation, that arising from energy conservation, by imposing also an upper limit Q_1 on the k_T integration. Before the imposition of this upper limit, the $x^{-0.5}$ term is in fact accompanied by a multiplicative logarithmic factor. With the upper limit, it is transformed into a sum of pure powers x^{-N} . The two leading values of N are

$$-0.5(1 - \Delta^2), \quad -0.5(1 - 4\Delta^2)$$

where $\Delta = [\pi/\log(Q_1/Q_0)]$. There are an infinite number of such powers, and as $Q_1 \rightarrow \infty$ they all coalesce at $x^{-0.5}$. However, with $Q_0 = 1$ GeV and Q_1 the maximum γ^*p energy attainable at HERA, the leading power is only $x^{-0.35}$. Thus the energy-conservation corrections to the Lipatov equation are substantial. This has recently been verified in the framework of the exact Lipatov equation by Forshaw, Harriman and Sutton^[16], while Bartels and Lotter^[17] have confirmed the important effects arising from the lower limit Q_0 .

5. Rapidity-gap physics

Figure 8 depicts the process known in proton-proton or proton-antiproton collisions as diffraction dissociation, in which one of the incoming hadrons emerges with only very small change of momentum. A very small fraction ξ , less than 5% or so, of its initial momentum is supposed to have been carried off by a pomeron, which collides with the other hadron. The cross section is of the form^[18]

$$\frac{d^2\sigma}{dt d\xi} = F_{P/p}(t, \xi) \sigma^{Pp} \quad (10)$$

where t is the momentum transfer from the initial to the final fast proton and $F_{P/p}$ is the probability for it to have “radiated” a pomeron:

$$F_{P/p}(t, \xi) = \frac{9\beta^2}{4\pi^2} [F_1(t)]^2 \xi^{1-2\alpha(t)} \quad (11)$$

with $\beta^2 \approx 3.5 \text{ GeV}^{-2}$ and $F_1(t)$ the elastic form factor of the proton. It is not essential to insist that the fast proton reaches the final state without breaking up; if one does not, then

one should omit the elastic form factor. The quantity σ^{Pp} is, by definition, the cross section for the pomeron interacting with the other proton and is the object of interest. (It should be noted that some authors adopt a definition of $F_{P/p}$ that differs from mine by a factor $\pi/2$, with a corresponding adjustment to σ^{Pp}).

Because there is a very fast proton (whether or not it has been allowed to break up), there is a rapidity gap in the final state; that is just kinematics. Note, however, that the presence of the gap is not sufficient to guarantee that pomeron-exchange is involved. There is good evidence that often instead it is f, ρ, \dots or even a pion. In order to check that there is no such contamination, one needs to check the presence of the factor $\xi^{1-2\alpha(t)}$ with $\alpha(t)$ the pomeron trajectory (4), rather than with the ρ, ω, f, a trajectory of figure 1 or the pion trajectory. In order to minimise the contamination^[19], ξ should be as small as possible, but t should not be too small (though not more than about 1 GeV², so that one may reasonably expect that only a single pomeron is being exchanged).

Some part of the pomeron-proton cross section σ^{Pp} is expected to be contributed from events with high p_T jets. That this is so has been verified by the UA8 experiment at the CERN collider, which finds^[20] events with $p_T > 8$ GeV/c jets. To calculate the cross-section expected for such events, one needs to know the pomeron structure function^[21]. We know that the pomeron is exchanged between quarks, since pomeron exchange generates the total hadron-hadron cross sections shown in figure 3. Further, it seems to couple to single quarks: the evidence for this is that^[22] the $s^{0.08}$ term in $\sigma^{\text{TOT}}(\pi p)$ is close to 2/3 its magnitude in $\sigma^{\text{TOT}}(pp)$. Thus, even though we believe that what is exchanged is gluons, they are coupling to quarks when they generate the pomeron. This led Donnachie and me to suppose^[18] that the pomeron structure function is dominantly quark; unlike most other authors, we believe that its gluon component is rather small. Hence our diagram for the UA8 process is that of figure 10a, where one of the quarks to which the pomeron couples participates in the hard scattering that produces the high- p_T jets, and the other is a longitudinal spectator. We find that the quark structure function of the pomeron is, for each light quark or antiquark,

$$x q_P(x) = \frac{1}{3} C \pi x (1 - x) \quad (x \geq 0.1) \quad (12)$$

where C is the coefficient of $x^{-0.08}$ in the corresponding total proton quark distribution. Note that x is the Bjorken fractional-momentum variable relative to the momentum of the pomeron. The fit to the data shown in figure 7 corresponds to $C = 0.23$, a somewhat larger value than the value we originally used^[18], which came from EMC data. If we sum over all the quarks, we obtain $x(1 - x)$ with multiplying coefficient just a little greater than 1.

The UA8 data^[20] seem to be in agreement with this, though it is not possible to determine from those data whether the pomeron structure function is quark or gluon. But the data do rule out the possibility that the structure function saturates a momentum sum rule, which was expected by some authors; as the pomeron is not a state, but only something that is exchanged, there is no reason to have saturation.

An interesting feature of the UA8 data is that in a significant fraction of events all, or nearly all, the energy of the pomeron-proton collision is taken by the pair of high- P_T jets. The natural explanation^[23] is that these events are generated not by a process of the type of figure 9a, but rather figure 9b, which has no longitudinal spectator jet coupled to the pomeron.

Important information about the pomeron structure function will come from HERA, since γ^* -pomeron collisions probe directly the quark structure function. With our structure function (12), at small x some 10% of the total events that make up the total νW_2^p should be expected^[18] to have a very fast proton. One can also look for the analogue of the events corresponding to figure 9b, namely fast-proton events with two high- P_T jets and nothing else. One can use real photons to look for these, and the cross-section should be quite large^[23], $\sigma^{\gamma p} \approx 1 \text{ nb}$ for $p_T > 5$ GeV/c.

6. Conclusions

Pomeron physics began more than 30 years ago and is nowadays a very active and interesting area of study. The phenomenology of the soft pomeron is surprisingly simple and has allowed several successful predictions. Soft pomeron exchange is nonperturbative gluon exchange, or maybe glueball exchange. There is some understanding of the theory behind this, but this is a difficult subject and further work on it will need to be guided by more data, such as will come from HERA. Finally, the possibility that HERA will discover also the Lipatov pomeron, which corresponds to perturbative gluon exchange, opens up a new opportunity for studying aspects of QCD.

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Figure 1 The ρ, ω, f, a trajectory .

Figure 2 Reggeon exchange. The jagged line represents a sum of particle exchanges

Figure 3 Total cross-sections

Figure 4 ISR data for pp and Tevatron data for $\bar{p}p$ elastic scattering, with 1985 curves from reference 3 .

Figure 5 Mechanism for high- p_T jet production: (a) central hard scattering and two structure functions; (b) a ladder model – the circles represent the structure functions .

Figure 6 The parton model for νW_2

Figure 7 NMC data for νW_2 , with fit of the form (8) .

Figure 8 Diffraction dissociation .

Figure 9 (a) Diffractive high- p_T jet production; (b) an additional mechanism